

IR: Evaluation

Evaluation of IR systems

In **ad hoc** document retrieval, the system is given a short query q and the task is to produce the best ranking of documents in a corpus, according to some standard metric such as average precision (AP).

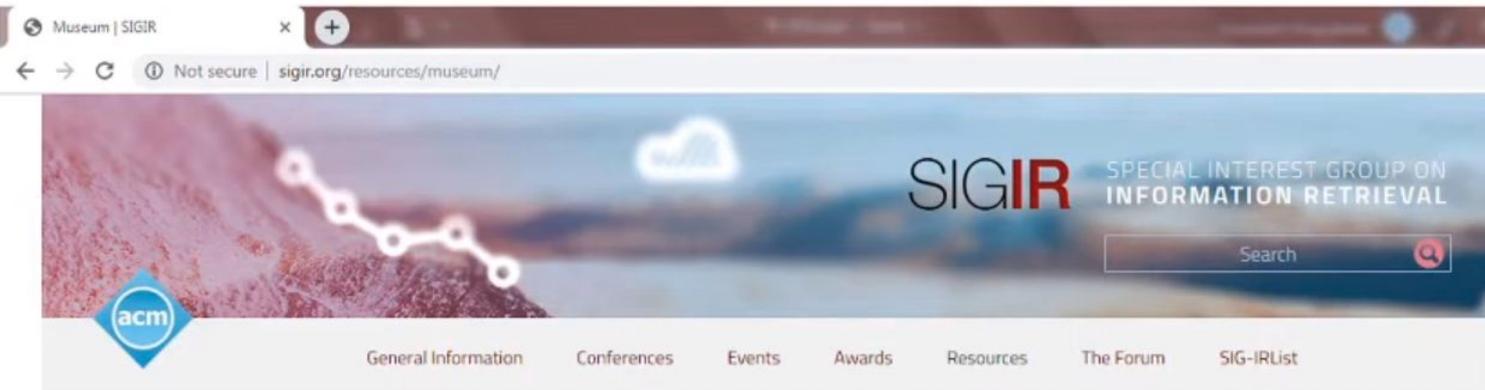


Earlier we had drop-downs for query field. Nowadays, query is a free-text!

Standard Test collections for Ad-Hoc retrieval

- Cranfield collection [1950]: Contains 1398 abstracts of journal articles, 225 queries, exhaustive judgements for all query document pairs.
- Text Retrieval Conference (TREC) [1992]: 1.89 billion documents , relevance judgments for 450 information needs. Judgements for top-k documents.
- GOV2: 25 million .gov web pages!
- NTCIR and CLEF: Cross language information retrieval collection has queries in one language over a collection with multiple languages.
- Reuters-RCV1, 20 Newsgroups, ...

The SIGIR Museum



Museum

**Report on the first stage of an investigation
onto the comparative efficiency of indexing
systems**

Cyril W. Cleverdon The College of Aeronautics, Cranfield, England, 1960

A Rich Area for Research

SIGIR 2018

Session 5C: New Metrics

-     Leif Azzopardi, Paul Thomas, Nick Craswell:
Measuring the Utility of Search Engine Result Pages: An Information Foraging Based Measure. 605-614
-     Fan Zhang, Ke Zhou, Yunqiu Shao, Cheng Luo, Min Zhang, Shaoping Ma:
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-     Enrique Amigó, Damiano Spina, Jorge Carrillo de Albornoz:
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SIGIR 2017

Session 1A: Evaluation 1

-     Gordon V. Cormack, Maura R. Grossman:
Navigating Imprecision in Relevance Assessments on the Road to Total Recall: Roger and Me. 5-14
-     Ye Chen, Ke Zhou, Yiqun Liu, Min Zhang, Shaoping Ma:
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-     Tetsuya Sakai:
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-     Xiaolu Lu, Alistair Moffat, J. Shane Culpepper:
Can Deep Effectiveness Metrics Be Evaluated Using Shallow Judgment Pools? 35-44

Evaluation

How to compare Search Engines? How good is an IR system?

- Various evaluation methods
 - **Precision/Recall**
 - Mean Average Precision
 - Mean Reciprocal Rank
 - If first relevant doc is at kth position, $RR = 1/k$.
 - NDCG
 - Non-Boolean/Graded relevance scores
 - $DCG = r_1 + r_2/\log_2 2 + r_3/\log_2 3 + \dots r_n/\log_2 n$

Precision

Precision measures how many of the retrieved documents are actually relevant.

- Precision focuses on the quality of results.
- High precision means fewer false positives.

Example:

If you retrieve 10 documents, and 8 of them are relevant, the precision is $8/10=0.8$.

Recall

Recall measures how many of the relevant documents were successfully retrieved.

Recall focuses on completeness of results.

High recall means fewer false negatives.

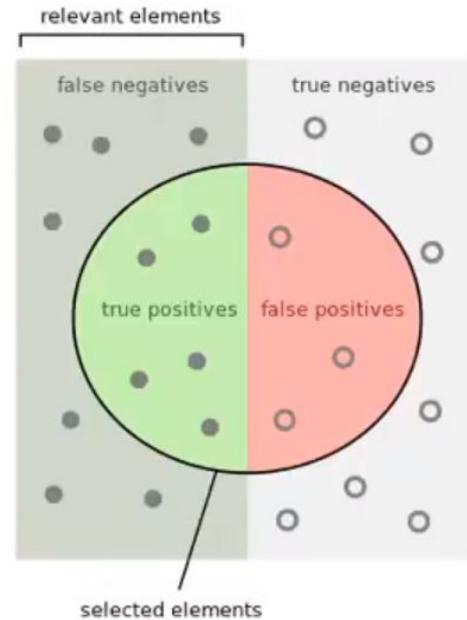
Example:

If there are 20 relevant documents and you retrieved 15 of them, the recall is $15/20=0.75$.

Precision and Recall

$$\text{precision} = \frac{\begin{matrix} \text{(True Positives)} \\ |\{\text{relevant documents}\} \cap \{\text{retrieved documents}\}| \\ \hline |\{\text{retrieved documents}\}| \\ \text{(True Positives + False Positives)} \end{matrix}}{\text{recall} = \frac{\begin{matrix} \text{(True Positives)} \\ |\{\text{relevant documents}\} \cap \{\text{retrieved documents}\}| \\ \hline |\{\text{relevant documents}\}| \\ \text{(True Positives + False negatives)} \end{matrix}}$$

Image Source: Wikipedia



How many selected items are relevant?

$$\text{Precision} = \frac{\text{green}}{\text{green} + \text{red}}$$

How many relevant items are selected?

$$\text{Recall} = \frac{\text{green}}{\text{green} + \text{dark green}}$$

Precision and Recall

- An IR system retrieves the following 20 documents.
- There are 100 relevant documents in our collection.
- Hollow squares represent irrelevant documents.
- Solid squares with 'R' are relevant.

	R	R		R			R		
			R	R	R	R			

- What is Precision?
- What is Recall?

Precision and Recall

- An IR system retrieves the following 20 documents.
- There are 100 relevant documents in our collection.
- Hollow squares represent irrelevant documents.
- Solid squares with 'R' are relevant.

	R	R		R			R		
			R	R	R	R			

- What is Precision? Precision = $8/20$.
- What is Recall? Recall = $8/100$.

...

Precision is concerned with minimizing false positives.

Recall is concerned with minimizing false negatives.

When True Negatives Matter

True negatives are useful in other evaluation metrics, like:

- **Accuracy:**

Accuracy = $\frac{TP+TN}{\text{Total Documents}}$

Accuracy considers **all outcomes**, including true negatives. However, it can be misleading in imbalanced datasets (many irrelevant documents).

- **Specificity (True Negative Rate):**

Specificity = $\frac{TN}{TN+FP}$

Specificity measures how well the system avoids retrieving irrelevant documents. It's less relevant for information retrieval tasks but useful in other fields like medical testing.

Can we do better?
Can we have one number to express quality?

F-Measure

- One measure of performance that takes into account both recall and precision.
- Harmonic mean of recall and precision:

$$F = \frac{2PR}{P + R} = \frac{2}{\frac{1}{R} + \frac{1}{P}}$$

Why the Harmonic Mean?

- The harmonic mean penalizes low values.
- If either Precision or Recall is very low, the F1-Score will also be low.
- This ensures that **both Precision and Recall** are balanced. A system cannot perform well overall by excelling in just one of the two metrics.

Arithmetic Mean

- What is the arithmetic mean of:
 - 1,2,3
 - 1,2,3,4,5
 - 1,2,3,4,5,6,7
- What is the arithmetic mean of:
 - 1 ... 99

$$\text{Answer: } \frac{1}{n} \sum_{n=1}^{99} n = \frac{1}{n} \cdot \frac{n(n+1)}{2} = \frac{99 \cdot 100}{2} = 4950$$

Arithmetic Mean

- What is the arithmetic mean of:
 - 7,8,9 ?
 - 11,13,15?
- What is the arithmetic mean of:
 - 1, 9, 10
 - 6.7
 - 1, 8, 10
 - 6.3
 - 1, 7, 10
 - 6

Geometric Mean

- What is the geometric mean of 2 and 8 ?
- Answer: $\sqrt{2 \cdot 8} = \sqrt{16} = 4$. (Arithmetic Mean is $\frac{2+8}{2} = 5$.)

$$\left(\prod_{i=1}^n x_i \right)^{\frac{1}{n}} = \sqrt[n]{x_1 x_2 \cdots x_n}$$

Geometric Mean

- What is the geometric mean of:
 - 7,8,9 ? AM=8, GM=7.96
 - 11,13,15? AM=13, GM=12.89
- What is the geometric mean of:
 - 1, 9, 10
 - AM=6.7, GM=4.48
 - 1, 8, 10
 - AM=6.3, GM=4.31
 - 1, 7, 10
 - AM=6, GM=4.1

Quiz

Which computer will you prefer?

	Computer A	Computer B	Computer C
Program 1	1	10	20
Program 2	1000	100	20

Time taken by two programs to execute on different computers.

Quiz

Which computer will you prefer?

	Computer A	Computer B	Computer C
Program 1	1	10	20
Program 2	1000	100	20

	A	B	C
Prg. 1	1	10	20
Prg. 2	1	0.1	0.02
A. Mean	1	5.05	10.01
G. Mean	1	1	0.63

	A	B	C
Prg. 1	0.1	1	2
Prg. 2	10	1	0.2
A. Mean	5.05	1	1.1
G. Mean	1	1	0.63

	A	B	C
Prg. 1	0.05	0.5	1
Prg. 2	50	5	1
A. Mean	25.03	2.75	1
G. Mean	1.581	1.58	1

Geometric Mean gives a consistent ranking
for normalized values.

Harmonic Mean

- What is the harmonic mean of 2 and 8 ?
- Answer: $\frac{2}{\frac{1}{2} + \frac{1}{8}} = 3.2$

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}}$$

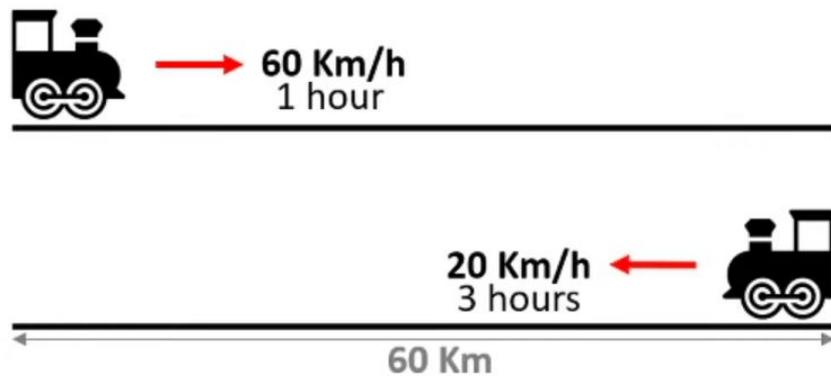
Harmonic Mean

- What is the harmonic mean of:
 - 7,8,9 ? AM=8, GM=7.96, HM=7.92
 - 11,13,15? AM=13, GM=12.89, HM=12.79
- What is the harmonic mean of:
 - 1, 9, 10
 - AM=6.70, GM=4.48, HM=2.48
 - 1, 8, 10
 - AM=6.30, GM=4.31, HM=2.45
 - 1, 7, 10
 - AM=6.00, GM=4.10, HM=2.41

Harmonic mean is more conservative than geometric mean and arithmetic mean

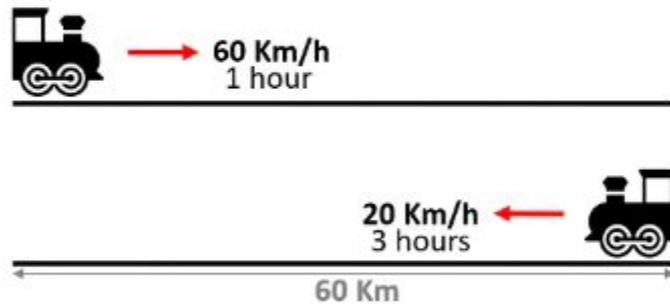
Quiz

- Can you compute the average speed?



Quiz

- Can you compute the average speed?



Compute AM, GM and HM of 60 and 20

AM = 40, GM = 63.25, HM = 30

Why harmonic mean for Precision and Recall?

Harmonic Mean

- What is the harmonic mean of:
 - 7,8,9 ? AM=8, GM=7.96, HM=7.92
 - 11,13,15? AM=13, GM=12.89, HM=12.79
- What is the harmonic mean of:
 - 1, 9, 10
 - AM=6.70, GM=4.48, HM=2.48
 - 1, 8, 10
 - AM=6.30, GM=4.31, HM=2.45
 - 1, 7, 10
 - AM=6.00, GM=4.10, HM=2.41

1. For F1 score to go up, we also need precision and recall to go up.
2. Dealing with the ratios

Precision and Recall

F1-Score **A Mean for Precision and Recall**

$$F_1 = \frac{2PR}{P+R}$$

simple harmonic mean of
precision and recall.

A more generalized formula:

$$F_\beta = (1 + \beta^2) \cdot \frac{\text{precision} \cdot \text{recall}}{(\beta^2 \cdot \text{precision}) + \text{recall}}$$

See "The truth of the F-measure" for a detailed discussion.

<https://www.toyota-ti.ac.jp/Lab/Denshi/COIN/people/yutaka.sasaki/F-measure-YS-26Oct07.pdf>

Compute Precision and Recall

- Case 1: First retrieval system.

1	2	3	4	5	6	7	8	9	10
	R	R		R			R		
			R	R	R	R			

- Case 2: Second retrieval system.

R	R	R	R	R	R	R	R		

20 documents retrieved. Assume that there are 100 relevant documents.



Compute Precision and Recall

Which retrieval system is bad and why?

- Case 1: Precision = $8/20$, Recall = $8/100$

	R	R		R			R		
			R	R	R	R			

- Case 2: Precision = $8/20$, Recall = $8/100$

R	R	R	R	R	R	R	R		

Treats this two system equally. If we take precision and recall.

Which IR system will you prefer? Can we do better?

Can we do better for ranked documents?

- Precision recall and F measure are set based measures.

Precision@k

- We cut-off results at k and compute precision.

	R	R		R			R		
			R	R	R	R			

- $P@1 = 0$
- $P@2 = \frac{1}{2}$
- $P@3 = \frac{2}{3}$
- $P@4 = \frac{2}{4}$



Disadvantage: If there are only 4 relevant documents in entire collection, and if we retrieve 10 documents, max precision achievable is only 0.4.

Recall@k

- Assume that there are 100 relevant documents.

	R	R		R			R		
			R	R	R	R			

- $R@1 = 0$



- $R@2 = 1/100$



- $R@3 = 2/100$



- $R@4 = 2/100$



Interpolated Precision

- We cut-off results at k^{th} **relevance** level.

	R	R		R			R		
			R	R	R	R			

- (Interpolated) $P@1 = 0.5$

	R
--	---

- (Interpolated) $P@2 = 2/3$

	R	R
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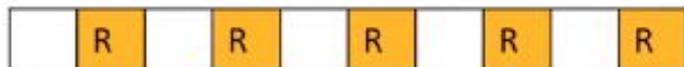
Interpolated Average Precision = $(0.5 + 0.66) / 2 = 0.58$

(if we are only interested in 2 levels of relevance)

*Interpolated precision at 0 is 1!

What is the Average Precision?

- Case 1:



- Average of Precision at each relevance level.

- Average Precision = $\frac{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}}{5}$

- Case 2:



- Average Precision = ?

For convenience, we refer to Interpolated Average Precision when we say AP

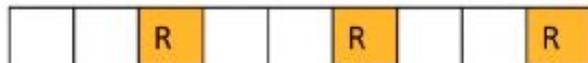
What is the Average Precision?

- Case 1:



- Average Precision = $\frac{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}}{5}$
- If there were 10 relevant documents, and we retrieved only five,
 - AP (at relevance level of 10) = $\frac{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 0 + 0 + 0 + 0 + 0}{10}$

- Case 2:



- What is AP at relevance level of 4? Assume there were 6 relevant documents in our collection.
 - AP = $\frac{1/3 + 1/3 + 1/3 + 0}{4}$

Mean Average Precision

MAP computes Average Precision for all relevance levels for a set of queries.

$$\text{MAP}(Q) = \frac{1}{|Q|} \sum_{j=1}^{|Q|} \frac{1}{m_j} \sum_{k=1}^{m_j} \text{Precision}(R_{jk})$$

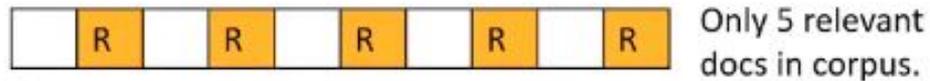
- MAP is a standard metric in **text retrieval** systems to evaluate the ranking of retrieved documents.
- Systems like **Apache Lucene**, **Elasticsearch**, or academic TREC (Text REtrieval Conference) use MAP to benchmark performance.

Example:

Given a dataset of queries and a set of documents (relevant and irrelevant), MAP determines how well the system ranks the relevant documents at the top.

Compute MAP

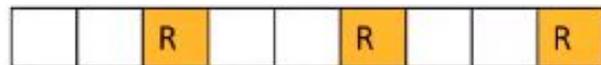
- Query1:



- Query2:



- Query3:

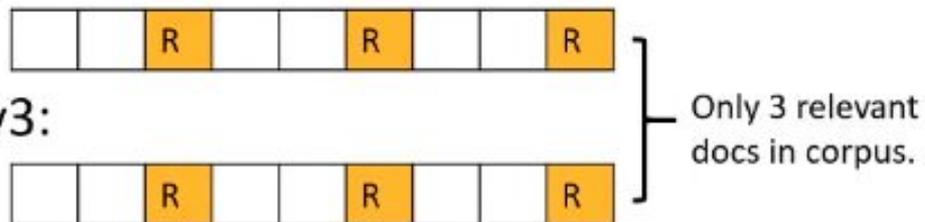


Compute MAP

- Query1:



- Query2:



- Query3:



- Compute MAP.

$$\text{MAP} = (1/2 + 1/3 + 1/3)/3$$

Quiz

- Can you compute MAP if you do not know the total number of relevant results for any given query?
 - No! This is the case with web search. Judges may not know how many relevant documents exist.

**How to compare two systems, if
results are ranked and graded?**

and we do not know the total number of relevant documents

Discounted Cumulative Gain (DCG)

DCG is designed to evaluate:

Ranking quality: The position of relevant items matters; higher-ranked items are given more importance.

Graded relevance: DCG accounts for relevance scores that may vary in degree (e.g., highly relevant, somewhat relevant, irrelevant).

Unlike precision and recall, which treat relevance as binary (relevant or not), DCG allows for a graded relevance score.

Discounted Cumulative Gain

$$DCG_k = \sum_{r=1}^k \frac{rel_r}{\log(r+1)}$$

DCG_k = DCG at position k

r = rank

rel_r = graded relevance of the result at rank r

DCG Example

- Presented with a list of documents in response to a search query, an experiment participant is asked to judge the relevance of each document to the query. Each document is to be judged on a scale of 0-3 with:
 - 0 → not relevant,
 - 3 → highly relevant, and
 - 1 and 2 → "somewhere in between".

DCG Example

- Compute DCG

i	rel_i	$\log_2(i + 1)$	$\frac{rel_i}{\log_2(i + 1)}$
1	3	1	3
2	2	1.585	1.262
3	3	2	1.5
4	0	2.322	0
5	1	2.585	0.387
6	2	2.807	0.712

$$DCG_6 = \sum_{i=1}^6 \frac{rel_i}{\log_2(i + 1)} = 3 + 1.262 + 1.5 + 0 + 0.387 + 0.712 = 6.861$$

Which system is better?

- 3,3,3,2,2,2 or 3,2,3,0,1,2 ?

Which system is better?

- 3,3,3,2,2,2 or 3,2,3,0,1,2 ?

Results from System 1				Results from System 2		
rel_i	$\log_2(i+1)$	$\frac{rel_i}{\log_2(i+1)}$		rel_i	$\log_2(i+1)$	$\frac{rel_i}{\log_2(i+1)}$
3.00	1.00	3.00		3.00	1.00	3.00
3.00	1.58	1.89		2.00	1.58	1.26
3.00	2.00	1.50		3.00	2.00	1.50
2.00	2.32	0.86		0.00	2.32	0.00
2.00	2.58	0.77		1.00	2.58	0.39
2.00	2.81	0.71		2.00	2.81	0.71
		8.74				6.86

...

How DCG Works

- Items ranked **higher** contribute more to the overall score due to the logarithmic discount.
- The relevance score rel_i reflects how useful or important an item is.

The logarithmic discount means:

- Rank 1 has **no discount** (most important).
 - Rank 2 gets a **logarithmic penalty**, and so on.
- Thus, DCG favors relevant items appearing earlier in the ranked list.

Which system is better?

- 3,2,3,0,1,2 or

- 3,3,3,2,2,2,1,0

What if there are unequal number of documents?



- Ideal DCG at 6 is (the best value) DCG for 3,3,3,2,2,2
- Normalize DCG with Ideal DCG value.
- NDCG for System 1 = $DCG/IDCG = 0.785$.
- NDCG for System 2 = 1.

For a set of queries Q , we average the NDCG.

...

Normalized DCG (NDCG)

To make DCG comparable across queries or datasets, we normalize it by the **ideal DCG (IDCG)** — the best possible DCG score if all relevant items were ranked in perfect order.

$$\text{NDCG}_p = \text{DCG}_p / \text{IDCG}_p$$

- **IDCG** is the DCG when the most relevant items are sorted in perfect order.
- **NDCG** ranges from 0 (worst ranking) to 1 (perfect ranking).

Applications of DCG

Search Engines: Evaluating how well search engines rank relevant documents (e.g., Google, Bing).

Recommender Systems: Evaluating the ranking of recommended items (e.g., Netflix, Amazon).

Learning to Rank: Machine learning models trained to optimize ranking systems.

Information Retrieval: Systems like Elasticsearch and Lucene use DCG to measure performance.

Why Use DCG?

Graded Relevance: Accounts for varying degrees of relevance (not just binary).

Position-Aware: Rewards systems for ranking relevant items earlier in the list.

Normalization (NDCG): Allows for comparisons across different queries or datasets.

Thank you for your time and attention.